Exercise 85

The average blood alcohol concentration (BAC) of eight male subjects was measured after consumption of 15 mL of ethanol (corresponding to one alcoholic drink). The resulting data were modeled by the concentration function

$$C(t) = 0.0225te^{-0.0467t}$$

where t is measured in minutes after consumption and C is measured in mg/mL.

(a) How rapidly was the BAC increasing after 10 minutes?

(b) How rapidly was it decreasing half an hour later?

Source: Adapted from P. Wilkinson et al., "Pharmacokinetics of Ethanol after Oral Administration in the Fasting State," *Journal of Pharmacokinetics and Biopharmaceutics* 5 (1977): 207–24.

Solution

To find how fast the concentration changes, take the derivative of $C(t)$.

$$\begin{aligned}
C'(t) &= \frac{d}{dt}[C(t)] \\
&= \frac{d}{dt}(0.0225te^{-0.0467t}) \\
&= 0.0225\frac{d}{dt}(te^{-0.0467t}) \\
&= 0.0225\left\{\left[\frac{d}{dt}(t)\right]e^{-0.0467t} + t\left[\frac{d}{dt}(e^{-0.0467t})\right]\right\} \\
&= 0.0225\left\{(1)e^{-0.0467t} + t\left[(e^{-0.0467t})\cdot\frac{d}{dt}(-0.0467t)\right]\right\} \\
&= 0.0225\left\{e^{-0.0467t} + t\left[(e^{-0.0467t})\cdot(-0.0467)\right]\right\} \\
&= 0.0225e^{-0.0467t}(1 - 0.0467t)
\end{aligned}$$

Evaluate it at $t = 10$ and $t = 30$.

$$C'(10) = 0.0225e^{-0.0467(10)}[1 - 0.0467(10)] \approx 0.00752$$

$$C'(30) = 0.0225e^{-0.0467(30)}[1 - 0.0467(30)] \approx -0.00222$$

At 10 minutes, the blood alcohol concentration is increasing by 0.00752 (mg/mL) per minute. At 30 minutes, the blood alcohol concentration is decreasing by 0.00222 (mg/mL) per minute.